

Superluminal Physics with Superconducting Circuit Technology

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We introduce a toolbox for the quantum simulation of superluminal motion with superconducting circuits. We show that it is possible to simulate the motion of a superconducting qubit at constant velocities that exceed the speed of light in the electromagnetic medium and the subsequent emission of Ginzburg radiation. We consider as well possible setups for simulating the superluminal motion of a mirror, finding a link with the superradiant phase transition of the Dicke model.

The fact that physical signals containing energy or information are not allowed to travel faster than the speed of light in vacuum is one of the best established facts in modern physics, and the cornerstone of one of its deepest principles, namely, causality. However, this does not prevent one from considering interesting features of the considered model at superluminal speeds. On the one hand, not all velocities are physical, in the sense that they do not need to carry any content of information. A related example can be found in Ref. [1]. On the other hand, experiments do not typically take place in vacuum, but in some form of medium, in which light moves at slower speeds. Therefore, it may be possible to consider physical motion at velocities exceeding those of light in the medium but not in vacuum. Classically, this gives rise to the well-known Cerenkov effect, where an oscillating electric charge generates classical electromagnetic radiation. In the quantum realm, the counterpart of the Cerenkov effect involves a neutral body or any sort of perturbation moving at superluminal speeds, i.e., the so-called Ginzburg radiation [2–5].

Quantum simulators are controllable quantum platforms aiming at reproducing the dynamics and static properties of complex quantum systems. They might soon be able to outperform classical computers when the analyzed quantum systems reach quantum supremacy. Quantum simulators can also be conceived as helpful tools which enhance our understanding of modern theoretical physics by allowing us to go beyond its fundamental laws. Along this vein, phenomena and effects which are not amenable to experiments due to technical or fundamental reasons, respectively, are now within reach of the burgeoning field of quantum simulations, ranging from magnetic monopoles [6] to traversable wormholes [7].

Superconducting circuits are one of the most promising quantum platforms for the development of scalable quantum technology and could be among the first in demonstrating quantum supremacy [8, 9]. In parallel, they are also a natural testbed for relativistic physics in quantum mechanics and quantum field theory, either in direct or simulated observations. For instance, the generation of photons out of the quantum vacuum due to the motion of mirror-like boundary conditions at relativistic speeds, namely, the dynamical Casimir effect (DCE), has been demonstrated in superconducting cir-

cuit architectures [10]. Along the same lines, a quantum simulation of the generation of acceleration radiation by means of the relativistic motion of a superconducting qubit has been theoretically proposed [11], i.e., cavity-enhanced Unruh effect. While the ultrafast variation of magnetic fluxes allows to achieve highly relativistic effective velocities, exploring both DCE and Unruh physics, breaking the light barrier with superconducting circuits, either in a medium or a quantum simulation, remains unexplored. Indeed, DCE experiments are restricted to velocities well below the light speed in the medium.

In this Letter, we introduce a toolbox for quantum simulation of superluminal motion with superconducting circuits. We will show that it is possible to simulate with current platforms the motion of a superconducting qubit at constant speeds exceeding the speed of light, even in vacuum, in the electromagnetic environment provided by a transmission line resonator. Remarkably, this effective superluminal motion can trigger the emission of Ginzburg radiation. We discuss as well the possibility of achieving superluminal constant velocities in the simulation of mirror-like boundary motion, which is out of reach of the current DCE setups. We find that an experimental setup similar to the one required for testing Dicke model physics in the thermodynamic limit can be used for the simulation of the Hamiltonian of a mirror moving at superluminal speeds. Moreover, we find a link with to Dicke superradiant phase transition in the thermodynamic limit. Notice that the emission of radiation by means of superluminal motion has at least two key differences with Unruh and DCE physics, namely that it only appears above the threshold of the speed of light in the medium and that it does not require accelerations.

Ginzburg radiation.— Let us start by showing how a qubit interacting with a single resonator mode via a quantum Rabi Hamiltonian emits radiation when moving at superluminal speeds, which can be understood as a particular case of Ginzburg radiation. This model can be described by the Hamiltonian

$$\mathcal{H} = \omega a^\dagger a + \frac{\omega_q}{2} \sigma_z + \mathcal{H}_I(x_q), \quad (1)$$

where ω_q is the qubit energy spacing, σ_z and σ_x are the usual Pauli operators acting on the qubit Hilbert space, and we consider $\hbar = 1$. We assume that the system dynamics effectively

involves a single resonator mode, described by annihilation and creation operators a and a^\dagger , respectively, of frequency $\omega = ck$ and wave vector $k = \pi/L$. Here, L is the resonator length and c is the speed of light, which in the case of a superconducting resonator is given by its electrical properties and takes a typical value of $c_0/3$, where c_0 is the speed of light in vacuum.

We can write the interaction Hamiltonian as

$$\mathcal{H}_I(x_q) = g \cos(kx_q) \sigma_x(a^\dagger + a), \quad (2)$$

where g is the coupling strength and x_q the qubit position [12]. Within perturbation theory, the probability of photon emission of a qubit starting in the ground state, with the field starting in the ground state as well, reads

$$P_e = g^2 \left| \int_0^T dt e^{i(\omega_q + \omega_0)t} \cos k x_q(t) \right|^2, \quad (3)$$

where ω_q , ω_0 are the frequencies of the qubit and the field respectively and $x_q(t)$ the, possibly time-dependent, position of the qubit. If the qubit is static this non-RWA probability is eventually negligible. However, if the qubit moves at constant velocity

$$x_q(t) = x_0 + vt \quad (4)$$

and assuming for simplicity $x_0 = 0$ we find:

$$P_e = g^2 \left| \int_0^T dt e^{i(\omega_q + \omega_0)t} \cos kv t \right|^2. \quad (5)$$

Therefore, if the velocity is:

$$v = \frac{\omega_q + \omega_0}{\omega_0} c, \quad (6)$$

the probability of photon emission is resonantly enhanced. Notice that this activation of the counterrotating terms of the Hamiltonian resembles the cavity-enhanced Unruh effect, a similarity first noted by Ginzburg in a more general context [3]. However, in this case there is no acceleration, and the effect comes from the superluminality of the qubit motion.

Indeed, let us highlight further analogies between the acceleration radiation scenario and the current constant-velocity one. If we consider an oscillatory motion starting at the center of the cavity and oscillating with frequency ω along the full resonator length [11] $x_q(t) = L/2 + L/2 \cos \omega t$, we can use the properties of the Bessel functions to write

$$\cos k x_q(t) \simeq -2J_1(\pi/2) \cos \omega t, \quad (7)$$

where $J_1(\pi/2)$ is the value of the Bessel function of the first kind evaluated at $\pi/2$. Using Eqs. (2) and (7), we find that the interaction Hamiltonian of this oscillatory motion would be

$$\mathcal{H}_I(x_q) \simeq -2gJ_1(\pi/2) \cos(\omega t) \sigma_x(a^\dagger + a), \quad (8)$$

which would be, as seen in Eq. (4), the same as the interaction Hamiltonian in the case of a trajectory with constant velocity $\frac{\omega}{\omega_0} c$ starting at $x = 0$ with a coupling strength $-2gJ_1(\pi/2)$. Therefore, we conclude that we can approximate a motion with constant velocity along the resonator by an oscillatory motion which starts at the center of the resonator and spans from mirror to mirror.

Putting all the above together, we find that it is possible to simulate superluminal constant velocities using existing experimental techniques. In this sense, the circuit QED architecture proposed in Ref. [11], the interaction Hamiltonian has the following dependence on the external magnetic flux,

$$\mathcal{H}_I(f) = g_0 \cos(f) \sigma_x(a^\dagger + a). \quad (9)$$

Here, $f = \frac{\phi}{\phi_0}$, where ϕ and ϕ_0 are the magnetic flux and flux quantum, respectively. Choosing the profile for the magnetic flux as

$$f = k x_q, \quad (10)$$

the Hamiltonians in Eqs. (2) and (9) are equivalent. Therefore, the modulation of the effective coupling constant mimics the motion of the qubit $x_q(t)$ inside the transmission-line resonator (TLR). In the case described above, this means that the magnetic flux profile

$$f = \frac{\pi}{2} + \frac{\pi}{2} \cos(\omega_q + \omega_0)t \quad (11)$$

implements an oscillatory motion around the center of resonator with frequency $\omega_q + \omega_0$, which would be equivalent to a motion with constant superluminal velocity $v = \frac{\omega_q + \omega_0}{\omega_0} c$ along the cavity, as shown above.

In the standard case in which the qubit is on resonance with the resonator, we will have a superluminal effective velocity $v = 2c$, which for typical circuit QED architectures would still be below the speed of light in vacuum. Adding a large detuning would enable the simulation of velocities that go even beyond the speed of light in vacuum.

In Fig. 1, we plot the results of numerical simulations. The dynamics is governed by a master equation where we introduce a cavity decay rate κ , a decay parameter Γ accounting for dissipative processes, as well as a decay Γ_φ for the dephasing of the qubits. The energy relaxation time and phase coherence time are denoted with $T_1 = 1/\Gamma$ and $T_2 = 1/\Gamma_\varphi$, respectively. We consider typical parameters in current experiments [13]. We see the excellent accuracy of the approximation in Eq. (7) and the neat resonance for the velocity $(\omega_q + \omega_0)/\omega_0$. In Fig. 1a the decay rate of the cavity is small and thus we observe perfect Rabi oscillations as expected from anti-Jaynes-Cummings dynamics [11]. In Fig. 1b, the decay rate is much larger, entering into the bad-cavity limit, which is also experimentally relevant [14]. In this case, the oscillations are washed out and the qubit is projected onto its excited state [15]. To retrieve the qubit states, one may use auxiliary resonators with dispersive microwave drivings to perform projective measurements of the qubits in the computational basis [16].

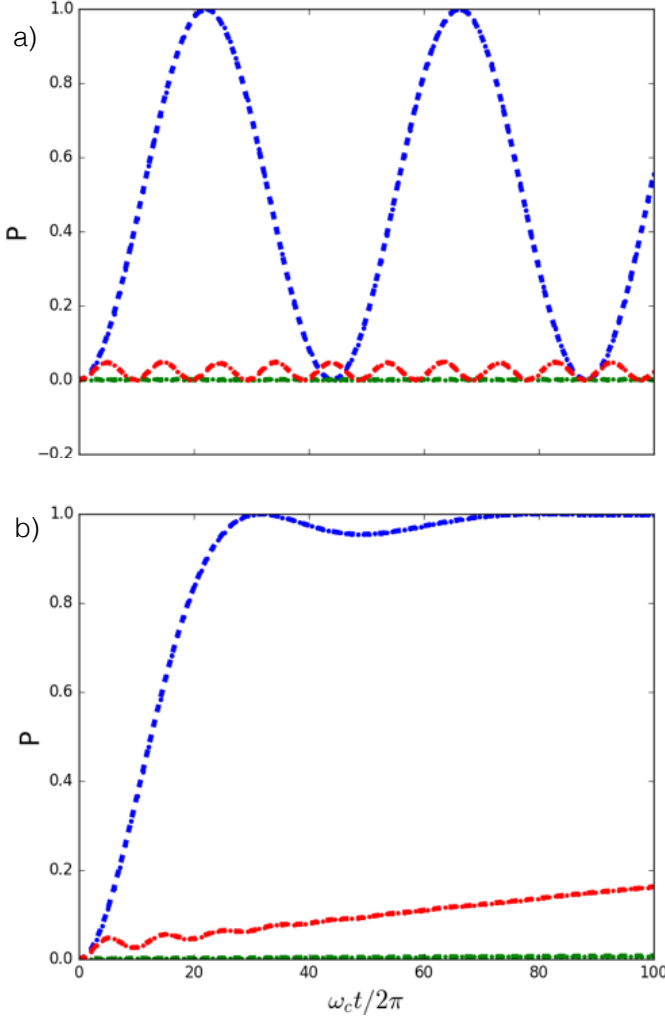


FIG. 1: (Color online) Probability of excitation for a qubit which is initially in the ground state and follows trajectories $x_q(t) = \frac{\pi}{2} + \frac{\pi}{2} \cos \omega t$ (dotted curves) and $x_q(t) = \frac{\omega}{\omega_0} c t$ (dashed curves), with $g = 0.02$ for the dotted curves and $g = -2 J_1(\pi/2) 0.02$ for the dashed ones. Dashed and dotted curves always superpose, showing the excellent accuracy of the approximation in Eq. (7). The frequencies are $\omega = 2\omega_0$ and $\omega_q = \omega_0/2$ (green), $\omega_q = 0.9\omega_0$ (red) and $\omega_q = \omega_0$ (blue). Therefore, in all cases $v = 2c$, but only the blue curves represent the resonance $\omega = \omega_q + \omega_0$. The qubit decay parameters are $\Gamma = 0.002$, and $T_2/T_1 = 0.67$, and we consider two cavity decay rates: a) $\kappa = 0.001$, and b) $\kappa = 0.1$ (bad-cavity limit), in units of ω .

A mirror moving at superluminal speeds.— Now we will consider a different scenario, where a mirror moves at superluminal speeds. It has been shown that the motion of optical boundaries [17, 18] or the perturbation of the refractive index [5] at constant and superluminal speeds generates photons out of the vacuum. This phenomenon somehow resembles the DCE, but it is radically different: there is no acceleration and it only appears at superluminal speeds. Moreover, it is also different from the Cerenkov effect, which requires the presence of a charge and is classical.

Although the DCE with oscillating motion is the most conspicuous example, other instances of boundary motion have been considered in the literature since the late 1960s [19–22]. However, to the best of our knowledge, the case of a mirror moving at superluminal speeds remains unexplored.

We consider now a 1D cavity of time-dependent length. In particular, let us assume that the cavity has a fixed length L until $t = 0$ and then the length changes in time, $L(t)$. The effective Hamiltonian for this system has been derived in Refs. [23, 24]:

$$H_{\text{eff}} = \sum_n \omega_n(t) \left(a_n^\dagger a_n + \frac{1}{2} \right) + \frac{\dot{L}(t)}{L(t)} \sum_n \sum_{j \neq k} (-1)^{n+j} \frac{jn}{j^2 - n^2} \sqrt{\frac{n}{j}} (a_n^\dagger a_j^\dagger + a_n^\dagger a_j - a_n a_j^\dagger - a_n a_j), \quad (12)$$

where

$$\omega_n(t) = \frac{\pi c n}{L(t)}, \quad (13)$$

and $\dot{L}(t)$ is the time derivative of $L(t)$.

Considering $L(t) = L(1 + \delta \sin \omega_d t)$ with $\delta \ll 1$, it is straightforward to realize that $\frac{\dot{L}(t)}{L(t)} \simeq v_{\text{max}} \cos \omega_d t$, which in the interaction picture leads to two-mode squeezing proportional to v_{max} if $\omega_d = \omega_k + \omega_j$. Therefore, the DCE is a particular case of the model embodied by Eq. (12).

However, the achievable mirror velocities in the celebrated circuit QED implementation of the DCE are severely limited [10]. In particular, the maximum velocity of the harmonic motion is $v_{\text{max}} = \delta L_{\text{eff}} \omega_d$ where δL_{eff} is the amplitude of the oscillation. On the one hand, ω_d is restricted by the fact that the SQUID needs to be operated well below its plasma frequency which typically means that $\omega_d < 20$ GHz. Indeed, it was of 10 GHz in Ref. [10]. On the other hand, the perfect SQUID-mirror equivalence only works if the condition $k_\omega L_{\text{eff}} \ll 1$ is met, that is L_{eff} must be smaller than the relevant wavelengths. Taking everything into account it turns out that $v_{\text{max}} \ll 2c$. Therefore, it is not possible to achieve the superluminal regime with the setup of Ref. [10].

Now, let us consider $L(t) = L - vt$. Note that even if $v < c$, this trajectory is completely unphysical, since it predicts an infinite acceleration at $t = 0$. Of course, this is not a concern in a simulated scenario. Using Eq. (12), we see that we have both two-mode squeezing and mode mixing proportional to

$$\frac{\dot{L}(t)}{L(t)} = \frac{v}{L - vt}. \quad (14)$$

Note the obvious restriction $vt < L$, which ensures $L(t) > 0$. However, we can consider that this is a restriction on time, not on velocity, and nothing prevents us from considering superluminal simulated velocities $v > c$. We can even restrict ourselves to shorter simulated times $vt \ll L$ where

$$\frac{\dot{L}(t)}{L(t)} = \frac{v}{L}. \quad (15)$$

Under this approximation, the Hamiltonian becomes time-independent. Note that c/L is the characteristic frequency scale of the system, so if we want velocities around c , the aim is to generate an interaction between the modes with a strength comparable to their frequencies, namely ultrastrong coupling among bosonic modes.

More specifically, let us particularise the Hamiltonian in Eq. (12) to the pair of lowest modes of a resonator, with frequencies ω_1, ω_2 where $\omega_2 = 2\omega_1 = 2\pi c/L$. Then, the coupling strength of the squeezing part of the Hamiltonian is

$$\Omega = \frac{\sqrt{2}}{3} \frac{v}{L}. \quad (16)$$

Thus, for $v = c$ we find $\Omega/\omega_1 \simeq 0.15$. Achieving this coupling strength and higher values in order to explore the superluminal region seems extremely challenging in a coupled-cavity setup, although it might be within reach in the case of SQUID-mediated coupling [25, 26].

Alternatively, we can also simulate one of the modes with an array of N qubits which are coupled to a single resonator mode. As is well-known from Dicke-model physics [27, 28], in the thermodynamic limit $N \gg 1$ the qubits are represented as well by a single collective bosonic mode. In this way, we can take advantage of the enhancement of the intermodal coupling $\Omega \propto \sqrt{N}$ [28, 29]. Indeed, the celebrated superradiant phase transition would take place at a critical value of the coupling $\Omega_c = \sqrt{\omega_1 \omega_2}/2 = \pi c/(\sqrt{2}L)$. This would correspond to a superluminal velocity $v = 3c/2\pi$, provided that we can neglect the diamagnetic term and any other extra terms [28, 30], which seems possible in some (e.g., Ref. [31]) but not all (e.g., Refs. [32, 33]) superconducting circuit architectures. In this way, we find a remarkable analogy between the Dicke superradiant phase transition and the physics of a mirror moving at relativistic speeds, which can be seen as an additional motivation for an experimental test of the Dicke model in the $N \gg 1$ limit with superconducting circuit technology. Notice that increasing the number N of qubits amounts to increasing the square of the simulated mirror velocity v^2 . So far, an array of 20 flux qubits coupled to a single resonator mode has been implemented in the laboratory, and the \sqrt{N} enhancement of the coupling has been proved up to 8 qubits [34].

In Fig. 2, we plot numerical simulations of the dynamics of the coupled two-mode model described above, including a cavity decay rate κ . Starting from an initial vacuum, we observe generation of photons for simulated superluminal velocities, well above the average number of thermal microwave photons at the 10 – 100 mK relevant regime of temperatures. To measure the number of photons in an implementation with superconducting circuits, one may employ standard circuit quantum electrodynamics techniques, e.g., dual path techniques [35].

Conclusions.— In summary, we have introduced a toolbox for the quantum simulation of superluminal motion with state-of-the-art superconducting quantum technology. We have shown that the achievement of simulated velocities exceeding the speed of light in the electromagnetic environment,

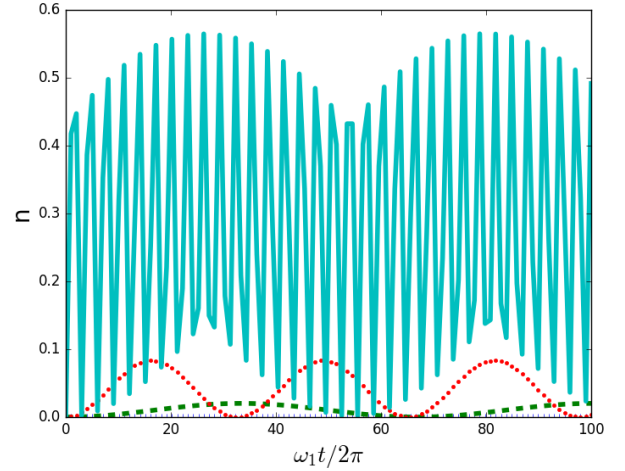


FIG. 2: (Color online) Total number of photons generated in the modes of frequencies ω_1 and ω_2 for a squeezing strength Ω given by Eq. (16) and $v/c = 0.1$ (dark blue, crosses), 1 (green, dashed), 2 (red, dotted) and $3\pi/2$ (light blue, solid). The latter corresponds to the critical value of the analogue superradiant phase transition. Note that $\Omega/\omega_1 \simeq 0.15 v/c$. We consider a decay rate $\kappa = 0.001$.

and possibly in vacuum, can be related to Unruh and DCE physics and, perhaps more surprisingly, to the superradiant phase transition of the Dicke model. In this way, we give an example of how cutting-edge quantum technologies can help not only to expand the frontiers of our technical abilities but also to explore the frontiers of theoretical physics.

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